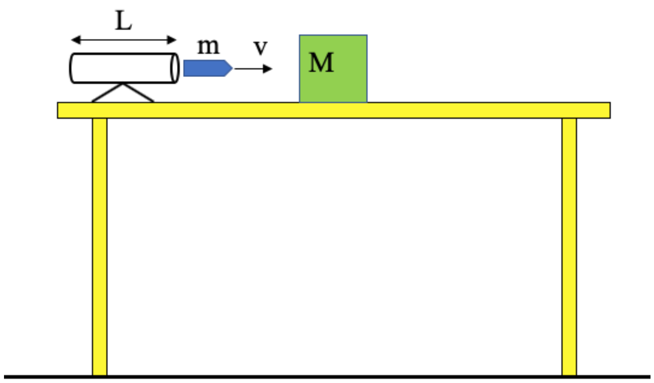
A dart of mass m moving through a tube of length L experiences a constant force F. After leaving the tube the dart strikes a box of mass M sitting on a table whose coefficient of friction is μ_k. The dart/block system then moves a distance D before coming to rest.



a.) Ignoring air resistance, and assuming all other parts of the experiment remains the same, if you want to maximize the stopping distance D, do you want the length of the tube to be long or short. Justify your response.

--large distance traveled before stopping requires a large exit velocity from the tube;
--the longer the tube, the more work force F will do;
--according to the work/energy theorem, the more work done, the greater the kinetic energy change;
--the greater the kinetic energy change, the greater the exit velocity, so apparently you want the tube to be as long as possible to effect the greatest stopping distance D.

b.) Someone derives an expression for the stopping distance to be $\frac{FLm}{(m+M)^2 \mu_k g}$. Without algebraic manipulation of equations, explain whether this equation accurately matches your explanation in Part a. Justify your response.

--it doesn't;
--this suggests the larger the accelerating force (which will produce a greater the exit velocity and is found in the numerator), the longer the stopping distance, which makes sense;
--this suggests the longer the tube over which F is applied (which will produce a greater the exit velocity), the longer the stopping distance, which makes sense;
--this suggests the smaller the coefficient of friction, being in the denominator, the greater the stopping distance, which makes sense;
--this suggests the bigger the total mass of the system (being in the denominator), the smaller the stopping distance, which makes sense;
--this suggests the bigger the dart's mass (being in the numerator), the larger the stopping distance, which probably isn't true (Fd=.5mv^2 . . . Fd is fixed, so the exit velocity is a function of Fd/m, and a big m will generate a small exit velocity . . . which generates a smaller stopping distance), which our expression doesn't suggest;
--this suggests that if the total mass of the system (m + M) gets bigger (it's in the denominator), D will be smaller . . . which is reasonable;
--the only reason the squaring of the (m + M) term in the denominator is reasonable is that a cursory look at the units requires that to be the case . . .

c.) The tube-force is changed so that it can be described as a function of position $F(x) = 12x$. Explain how this equation can be used to find the launch speed of the dart if the length of the tube is 2.0 meters.

--the work/energy theorem states that the net work done on the object equals the object's change of kinetic energy;
--its initial kinetic energy is zero and its final kinetic energy is .5mv^2;
--to determine the work done by a variable force, you have to determine the differential amount of work dW being done over a differential distance dx (that is, write out Fdx), then sum all those differential bits of work (via an integral) to get the total work done;
--with that, you can execute the work/energy theorem.

d.) Given the force function defined in Part c, what would happen to the launch velocity of the dart if the length of the tube was doubled. Justify your response.

--the limits of the integral would no longer be from zero to L but now would be from zero to 2L;
--the integral of xdx is .5x^2, so doubling the length of applied force would increase the work done by a factor of 4;
--according to the work/energy theorem, the work going up by 4 would affect the final kinetic energy, which is a function of v^2;
--the exit velocity would go up by a factor of 2 (tricky, eh?)

e.) What does the solution to Part d tell us about double the length of the tube in a situation in which the force was constant?

--apparently, doubling the length will double the work done, but because he kinetic energy is a function of v^2, the velocity won't double . . . it will increase by the square root of 2.

f.) It is claimed that the launch velocity is $v = \left(\frac{12L^2}{m}\right)$. Without algebraic manipulation of equation, explain whether this equations accurately matches your explanation of Part d. Justify your answer.

--it doesn't;
--this suggests that a large mass will produce a small exit velocity, which is reasonable;
--this suggests that a large L produces a large exit velocity, which is reasonable;

--what isn’t reasonable is that this relationship suggest that if you double the length, the velocity will increase by a factor of 4, whereas we deduced above that it should only increase by a factor of 2.